Alice and developments in mathematics in late 18th and early 19th century England : Pycior, Carroll, algebra and humour

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1 Introduction

In 1866, Lewis Carroll published his famous fantasy novel *Alice's adventures in Wonderland* followed in 1872 by *Through the looking glass, and what Alice found there*. Because Carroll was a pseudonym of the mathematician Charles Lutwidge Dodgson (1832-1898), mathematical interpretations have been given for the apparent nonsense in his *Alices*. In 2009, Melanie Bayley added the suggestion that the chapter about the mad tea-party was about the quaternions of Sir William Rowan Hamilton, but in an article called ALICE WITHOUT QUATERNIONS, which came online last September, I argue that that is not the case.

In her article Bayley mentioned that "the 19th century was a turbulent time for mathematics, with many new and controversial concepts," and she referred to the work of the historian Helena Pycior who had given mathematical interpretations of the *Alices*.[1] Having a bachelor's in physics I did know about Hamilton and quaternions[2] but I was not very familiar with the history of mathematics. Therefore, I searched for Pycior's work, trying to gain a better understanding of why and how these controversial concepts in mathematics had been important for the *Alices*.

It indeed appeared that in the 1980s Pycior had proposed that in his *Alices* Dodgson was mocking the new mathematics around him, which mainly consisted of the imaginary numbers, the negatives and symbolical algebra. Moreover, Pycior had found that in early nineteenth century England criticisms on mathematics had been given humorously. Showing that Dodgson could not accept some of the new developments in mathematics, she argued that through his *Alices* he also used humour to ridicule them.

In this unpublication I will give a summary of Pycior's views on algebra, humour and Dodgson's *Alices*, supplemented with a few remarks about Hamilton, as I encountered while working on the restoration of his private reputation. I hope that for non-mathematicians like me, or mathematicians who just are not familiar with this particular part of the history of mathematics, that will be as enlightening as it was for me. And I will abruptly stop where my article, ALICE WITHOUT QUATERNIONS, begins.

[1] Carroll wrote two Alice novels, the second one was published in 1872.

[2] In my 2017 essay, A Victorian Marriage : Sir William Rowan Hamilton, which was published in 2017, and the 2018 article A most gossiped about genius: Sir William Rowan Hamilton, I aimed to show that, contrary to what is generally assumed, Hamilton lived an, as he called it, 'studious and happy life'. In our 'gossip article', it was also discussed where the negative view on his private life came from.

2 Algebra in the early nineteenth century

In 1982 Pycior described algebra as it existed before 1800,

> in this algebra, frequently called universal arithmetic, letters stood for numbers or quantities, and the laws of arithmetic, such as commutativity of addition and multiplication, prevailed (Pycior 1982a, 150).

But things were changing rapidly, and

> by the middle of the nineteenth century [...] mathematicians had created many different algebras, including the quaternions (Ibid.).

Yet many people at first rejected the new algebras. What was difficult to accept about the quaternions of Sir William Rowan Hamilton (1805-1865) was their non-commutativity;[3] it was the first time that one of the laws of arithmetic appeared not to be universally true. Hamilton found the quaternions in 1843, and in 1852 he wrote to his friend Augustus De Morgan (1806-1871) about their reception:[4]

> I am not conscious of being on terms of even the slightest unkindness with a single Fellow of [Trinity College Dublin], and with several of them I am on a friendly and indeed affectionate footing. To be sure, many of them ridiculed me about the quaternions, and Charles Graves[5] once burst out with the exclamation, in my presence: 'It is astonishing what a prejudice exists against the quaternions, and that among people who confess that they know nothing of the subject!' 'Would it not be more discouraging', I replied, 'if the same prejudice continued among those who are acquainted with it?' (Graves 1882-89, III, 336).

Nevertheless, sometimes even for Hamilton it had been really hard; in 1853 he wrote to another friend:

> You will I hope bear with me if I say, that it required a certain capital of scientific reputation, amassed in former years, to make it other than dangerously imprudent to hazard the publication of [the *Lectures on Quaternions*, (Hamilton 1853)] which has, although at bottom quite conservative, a highly revolutionary air. It was a part of the ordeal through which I had to pass, an episode in the battle of life, to know that even candid and friendly people secretly, or, as it might happen, openly, censured or ridiculed me, for what appeared to them my monstrous innovations (Graves 1882-89, II, 682).

3 The symbolical approach to algebra

Already before Hamilton found the quaternions people had been stirring up the absolute truths of mathematics. Since the sixteenth century mathematicians had used negative and imaginary numbers, but around the 1800s these concepts still were not well understood, and therefore not accepted by everyone. Of imaginaries that seems easy to understand, but regarding the negatives it might appear odd. However, Pycior mentioned that textbooks then defined a negative number as a 'quantity obtained by the subtraction of greater from lesser

[3] Quaternions not being commutative means that there exist quaternions $\mathbf{p}$ and $\mathbf{q}$ for which $\mathbf{pq} \neq \mathbf{qp}$.
[4] Hamilton and De Morgan corresponded from 1841 until Hamilton's death in 1865.
[5] Charles Graves was a fellow mathematician and brother of Hamilton's later biographer Robert Graves.

quantities' or a 'quantity less than nothing' (Pycior 1984, 151); definitions which express the difficulty.

In 1830 George Peacock (1791-1858) published *A treatise on algebra*, about a symbolical approach to algebra (Peacock 1830). Pycior remarked,

> rather than define the negatives and imaginaries in a traditional fashion, Peacock chose to redefine algebra (Pycior 1984, 152).
> Like modern algebra, early nineteenth-century symbolical algebra admitted undefined entities; unlike modern algebra, symbolical algebra, in its original form, was governed by the laws of arithmetic (Pycior 1982a, 150).

Peacock's symbolical algebra, in which for instance the initially meaningless '×' could be used for both arithmetical multiplication and, for example, multiplication of line segments (Pycior 1984, 153), (Peacock 1830, 72-73, 84-87), introduced meaninglessness and arbitrariness, which many mathematicians found difficult to accept. In an 1982 article, 'Early criticism of the symbolical approach to algebra' (Pycior 1982b), Pycior cited an illustrative sentence from De Morgan,

> at first sight it appeared to us something like symbols bewitched, and running about the world in search of a meaning (Pycior 1982b, 395), (De Morgan A 1835a, 311),

a sentence already quite reminiscent of *Alice*.

Earlier in 1982 Pycior had published an article called 'Historical roots of confusion among beginning algebra students: A newly discovered manuscript' (Pycior 1982a). The first part of this title is explained by her remark that

> for the confused modern algebra student [...] solace might be derived [...] from the revelation that bewilderment and occasionally even rejection greeted the original formulation of the symbolical approach to algebra (Pycior 1982a, 150).

The second part of the title alludes to a very short spoof play she had found in 1979 and which is included in the article (Pycior 1982a, 153-156). The play, featuring De Morgan as the protagonist, was written as she argued by the mathematician William Frend (1757-1841) or his daughter Sophia Frend (1809-1892), or by both, between 1835 and 1838. The play spoofed De Morgan's 1835 *Elements of algebra* (De Morgan A 1835b), of which Pycior remarked that it was 'one of the first undergraduate algebra textbooks to incorporate the symbolical approach to algebra' (Pycior 1982a, 150).

4 Combining humour and mathematics

In her 1984 article Pycior connected Dodgson's *Alices* to 'a small but significant tradition of combining humor and mathematics' (Pycior 1984, 157) which had been established by the mid-nineteenth century, in which both Frend and De Morgan were actively involved, and of which the spoof play was an example.

William Frend's objections to the new mathematics had been fiercer than those of other mathematicians. Even though the concepts of negatives and imaginaries were not well understood, at the end of the eighteenth century most people had become used to working with them. But William Frend could not accept them; De Morgan wrote in his *A budget of paradoxes* that Frend, in his 1796 *The principles of algebra* (Frend 1796), made 'war of extermination upon all that distinguishes algebra from arithmetic' (De Morgan A 1872, 117).

As the 'earliest representative of the Victorian intersection of mathematics and humour' (Pycior 1984, 154), in 1803 Frend had been 'satirizing the mathematical concept of nothing' in an essay called 'Pantagruel's decision of the question about nothing' which ends, 'they who can make something out of nothing, shall have nothing to eat at the court of – Pantagruel' (Pycior 1984, 155), (Frend 1803, 113), (De Morgan A 1872, 124-125).

In 1827 De Morgan had befriended William Frend (De Morgan S 1882, 19), and knowing him well means that when in 1835 he published his *Elements of algebra* which contained the symbolical approach to algebra, he knew it would not be accepted by Frend. In the summer of 1835 De Morgan gave a copy of his *Elements* to Sophia Frend (Pycior 1982b, 395), and that may have led to the writing of the spoof play in which for instance, through the freedoms of symbolical algebra, based on the same assumptions a salary can vary by a factor of a million, and De Morgan's students use negative numbers to leave the course (Pycior 1982b, 402).

Augustus De Morgan and Sophia Frend married in 1837, within the time range given by Pycior for the origination of the spoof play, and it is easy to imagine that it was some sort of wedding gift; De Morgan will have enjoyed it. Hamilton remarked about his correspondence with De Morgan, 'I send him nonsense at times, and he sends me back wit in return, rising occasionally to humour.' Lady Hamilton was often amused by De Morgan's letters, but Hamilton did not always dare to show them to her, for instance when De Morgan joked about Paradise (Graves 1882-89, III, 44-45), (De Morgan A 1872, 118).

5 Humour, algebra and the *Alices*

Thus arguing that

> the eighteenth and nineteenth-century tradition of popular mathematics and mathematical humor formed the background to the work of not only Frend and De Morgan but Charles Dodgson as well (Pycior 1984, 158-159),

and emphasizing that, like Frend before him, also Dodgson did not accept symbolical algebra (Pycior 1984, 162-163), Pycior further suggested that especially symbolical algebra had inspired Dodgson to write his *Alices*;

> it was with visions of lines multiplied by lines, quantities less than nothing, and symbolical algebra dancing in his head that Dodgson first set about constructing an other-world (the underground) [6] in which meaninglessness and arbitrariness prevailed (Pycior 1984, 163).

This suggestion was underpinned by the observation that a letter, written in 1854, which showed 'Dodgson's struggle with the question of algebraic meaning' coincided in time with a mathematical reading party of which a fellow student later mentioned that 'it was there that *Alice* was incubated' (Pycior 1984, 161, 163). As another argument in support of the suggestion that symbolical algebra was connected to the *Alices*, Pycior drew attention to the fact that at least once Alice uses a phrase from De Morgan,

> echoing De Morgan's description of symbolical algebra, Alice declares: '*I* don't believe there's an atom of meaning in it' (Pycior 1984, 167), (Carroll 1866, 184). [7]

[6] The first version of the *Alices* was called 'Alice's Adventures under Ground'. For easy reading, I made a pdf version.

[7] The phrase comes from De Morgan's *Trigonometry and double algebra*, 'It is most important that the student should bear in mind that, with one exception [the sign =], no word nor sign of arithmetic or algebra has one atom of meaning throughout this chapter, the object of which is symbols, and their laws of combination' (De Morgan A 1849, 101).

But in his rejection of the then new mathematics Dodgson was not so extreme as Frend; contrary to Frend who called negatives 'absurd' (Frend 1796, 4), throughout his *Elementary treatise on Determinants* Dodgson did use negative numbers (Dodgson 1867). Yet even though he was used to working with negatives, he apparently still had trouble with the concept of 'quantities less than nothing'; Pycior argued that in his *Alices* Dodgson touched this problem several times, for instance at the mad tea-party where Alice meets the Hatter, the March Hare and the Dormouse, sitting at a table set for tea-time at six o'clock. (Carroll 1866, 95-111).

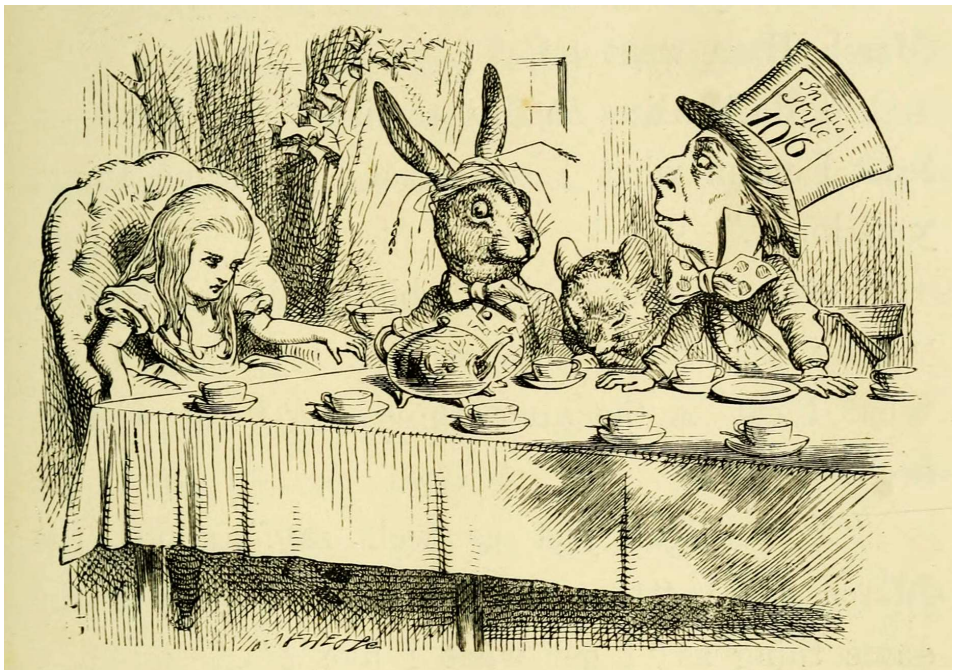

Figure 1: Alice at the mad tea-party

> The March Hare offers Alice 'more tea.' Alice replies that she has 'had nothing yet [and] so ... can't take more.' This provides the opportunity for the Hatter's comment: 'You mean you can't take less ... it's very easy to take more than nothing' (Pycior 1984, 164), (Carroll 1866, 106).

And here begins ALICE WITHOUT QUATERNIONS.

References

[Carroll 1866] Carroll, L, *Alice's adventures in Wonderland*, London: MacMillan and Co, 1866.
https://archive.org/details/alicesadventur00carr.

[Carroll 1872] Carroll, L, *Through the looking glass, and what Alice found there*, London: MacMillan and Co, 1872.
https://archive.org/details/throughlooking00carr.

[De Morgan A 1835a] De Morgan, A, Review of *A treatise on algebra* by George Peacock, *The Quarterly journal of education*, 9 (1835), 91-110, 293-311.
https://babel.hathitrust.org/cgi/pt?id=mdp.39015065413380&seq=103.

[De Morgan A 1835b] De Morgan, A, *Elements of algebra : preliminary to the differential calculus*, London: Taylor and Walton, 1835.
https://archive.org/details/elementsofalgebr00demouoft, 2nd ed. 1837.

[De Morgan A 1849] De Morgan, A, *Trigonometry and double algebra*, London: Taylor, Walton and Maberly, 1849.
https://archive.org/details/trigonometrydoub00demoiala.

[De Morgan A 1872] De Morgan, A, *A budget of paradoxes*, London: Longmans, Green, and Co., 1872.
https://archive.org/details/cu31924103990507.

[De Morgan S 1882] De Morgan, S, *Memoir of Augustus De Morgan*, London: Longmans, Green, and Co., 1882.
https://archive.org/details/memoirofaugustus00demouoft.

[Dodgson 1867] Dodgson, CL, *An elementary treatise on determinants*, London: MacMillan and Co, 1867.
https://archive.org/details/elementarytreati00carr.

[Frend 1796] Frend, W, *The principles of algebra*, London: GG and J Robinson, 1796.
https://archive.org/details/principlesalgeb00frengoog.

[Frend 1803] Frend, W, 'Pantagruel's decision of the question about nothing', *The Gentleman's Monthly Miscellany*, 1/1-5 (1803), 111-113.
https://babel.hathitrust.org/cgi/pt?id=hvd.32044089884662&seq=125.

[Graves 1882-89] Graves, RP, *Life of Sir William Rowan Hamilton, Knt. &c* vol I, II, III, Dublin: Hodges, Figgis, & Co., 1882, 1885, 1889.
https://archive.org/details/lifeofsirwilliam01gravuoft,
https://archive.org/details/lifeofsirwilliam02gravuoft,
https://archive.org/details/lifeofsirwilliam03gravuoft.

[Hamilton 1853] Hamilton, WR, *Lectures on quaternions*, Dublin: Hodges and Smith, 1853.
https://archive.org/details/lecturesonquater1853hami.

[Peacock 1830] Peacock, G, *A treatise on algebra*, Cambridge: J & JJ Dreighton, 1830.
https://babel.hathitrust.org/cgi/pt?id=ucm.5305745941&seq=5.

[Pycior 1982a] Pycior, HM, 'Historical roots of confusion among beginning algebra students: A newly discovered manuscript', *Mathematics Magazine*, 55/3 (1982), 150-156.
http://www.jstor.org/stable/2690081.

[Pycior 1982b] Pycior, HM, 'Early criticism of the symbolical approach to algebra', *Historia Mathematica*, 9/4 (1982), 392-412.
https://www.sciencedirect.com/science/article/pii/0315086082901057.

[Pycior 1984] Pycior, HM, 'At the intersection of mathematics and humor: Lewis Carroll's 'Alices' and symbolical algebra', *Victorian Studies*, 28/1 (1984), 149-170.
http://www.jstor.org/stable/3826762.