William Hamilton and the 'flaw' in Laplace, the flawed story about it, and William's proof

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1 The flawed story

Much has been said about Sir William Rowan Hamilton (1805-1865) having found in his youth an error in Laplace's *Mécanique Céleste*, which was the reason he attracted the attention of John Brinkley, Royal Astronomer of Ireland who lived at Dunsink Observatory, who then remarked, "This young man, I do not say *will be*, but *is*, the first mathematician of his age." This story is not true.

First, Graves unintentionally started the narrative; he had a tendency to neglect the general chronology of his Hamilton biography when he became emotional, indignant or in awe, which caused much confusion. Graves writes, "[Hamilton] signalised the beginning of this acquaintance [with Brinkley] with a great masterpiece by detecting a flaw in the reasoning by which Laplace demonstrates the parallelogram of forces." Yet Hamilton called his piece, given in the Appendix to the first volume of the biography, "Correction of an error of reasoning in Laplace's *Mécanique Céleste*." This seems to emphasise the "error," yet more likely emphasised the "correction" because William gave a new proof, which will be given hereafter. Calling it a "masterpiece" is clearly a happy exaggeration.

Second, the direct connection between the 'flaw' and Brinkley is not correct. Hamilton had found the 'flaw in Laplace's reasoning' on 31 May 1822, and a family friend, Mr. Kiernan, asked Hamilton to write it out so that he could show it to Brinkley, which he did. But it was not this "correction" which started his acquaintance with Brinkley, and indeed Graves called it the "seed" of their "personal acquaintance." On 31 October 1822 Hamilton wrote from Trim to Cousin Arthur that Mr. Kiernan told him, "I forgot your 'queries about Laplace' for a long time," queries Hamilton had also shown to his college tutor Boyton, but to which Graves comments that he was "not able to supply any information." Hamilton then wrote that Kiernan had continued, "but at last I laid them before Dr. Brinkley, who said he thought them ingenious, and he was so good as to say that he would write an explanation for you. He also desired me to bring you to him, and that he would be happy to know you, and to show you the Observatory. This of course, you know, is a great honour." Obviously, these queries were not the short proof Hamilton had given instead of Laplace's 'flaw' and which is shown hereafter; that certainly did not need an explanation of Brinkley. What seems likely is that Hamilton, who found the 'flaw' on page six of the Mécanique Céleste, thereafter worked himself through the rest of the book while writing the queries which attracted Brinkley's attention. When in the end he was invited to visit the Observatory, he brought with him two mathematical papers, by which Brinkley was impressed.

Third, although Brinkley was impressed by the value of these papers, he does not seem to have made the statement which is often repeated in this story, and was given above. This story has evolved in a similar way as the anecdotes in the gossip article; the sentence was most likely not a real quote, and it had no direct connection to Laplace. Without claiming here that the evolution of this story is extensively researched, it was found that the sentence was given by Augustus De Morgan in his 1866 obituary (p. 129) of Hamilton, recorded as an exaggerated expression of admiration. De Morgan continued by supposing that Brinkley did not say that; "we will undertake to say that he went no further than – "If the young man goes on as he has begun, he will become," &c. &c." Whether or not taken from De Morgan, in 1869 Clement Mansfield Ingleby gave the sentence as a fact, as having been uttered by Brinkley to a friend. In 1967 the error in Laplace was directly connected to Hamilton meeting Brinkley by Cornelius Lanczos, but he did not mention the sentence. In 1998 both meeting Brinkley and the sentence were given by MacTutor in connection to the error in Laplace, completing the story. De Morgan's doubts that Brinkley would have uttered such a firm statement seem to be corroborated by Maria Edgeworth, who wrote in August 1824 to her sister Honora that Hamilton was "an "Admirable Crighton" of eighteen;[1] a real prodigy of talents, *who, Dr. Brinkley says, may be a second Newton* – quite gentle and simple." The italics are either by her or by Graves.

2 Laplace's 'flaw' and Hamilton's proof

In the Appendix to the first volume of his biography Graves gives Hamilton's 'Correction of an Error of Reasoning in Laplace's 1799 *Mécanique Céleste*. The "reasoning" under scrutiny is in Laplace's first volume on p. 6, where he discusses the resultant force z of two perpendicular forces, x and y, with an angle θ between x and z. Quaternions and therefore vector analysis not yet having been found, Laplace uses the method of a parallelogram of forces, where the forces x and y are equal to the sides of the parallelogram, and the resultant force z is equal to its diagonal.

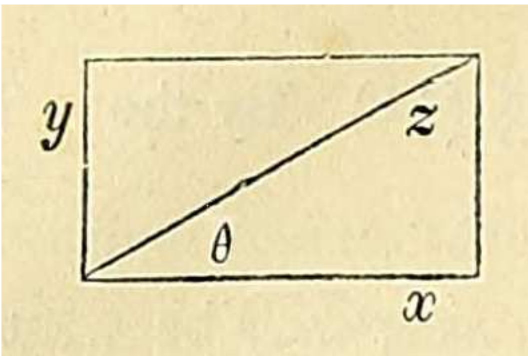

Figure 1: This diagram is constructed after the 1829 English translation of the *Mécanique Céleste* by Nathaniel Bowditch, p. 6 footnote 13a, in which Bowditch explains the use of the parallelogram of forces, "the forces x, y are equal to the sides of the parallelogram whose diagonal is z."

[1] Graves remarks, "Just 19 years."

Having introduced the notion of forces on p. 1 of the *Mécanique Céleste*, on the next page Laplace searches for the relation between x, z and θ, the two perpendicular forces x and y being given. On p. 6 he arrives at the trigonometric identity for the cosine of the still general angle $k\,\theta + \rho$, namely $x = z\cos(k\,\theta + \rho)$. He then shows, by taking $y = 0$ and thus $z = x$ and $\theta = 0$, that $\cos\,\rho = 1$, thus $\rho = 0$, therefore $x = z\cos\,k\,\theta$.

Laplace continues, as given in Bowditch' translation, that if on the other hand "we suppose $x = 0$, we shall have $z = y$, and $\theta = 1/2\,\pi$; $\cos k\,\theta$ being then equal to nothing, k ought to be equal to $2n+1$, n being a whole number, and in this case, x will be nothing whenever θ is equal to $(1/2\,\pi)\,/\,(2n+1)$; but x being nothing, we evidently have $\theta = 1/2\,\pi$; therefore $2n + 1 = 1$, or $n = 0$, consequently $x = z\cos\,\theta$."

This reasoning, leading to $n = 0$ and thus $k = 1$, is what Hamilton does not completely agree with. He remarks that for $\theta = 1/2\,\pi$ not only $x = 0$ and thus $\cos\,k\,\theta = 0$ if $k = 1$, but also if $k = 3, 5$, &c., and that seems to be his quite straightforward "correction". Interesting is that in parentheses Hamilton remarks that, where Laplace gives $2n + 1$ for k, "it might easily be shown to be of the form $4n + 1$." That would lead to $0 = z\cos((4n + 1)\,1/2\,\pi)$, and therefore $\cos 1/2\,\pi = 0$ modulo 2π, instead of modulo π as in case of $k = 2n + 1$. Thus skipping solutions for a negative value for y, Hamilton apparently regarded it as a physics problem for the given forces.

It indeed seems somewhat strange that Laplace first takes the trouble of defining $k = 2n+1$ and then just eliminates n instead of concluding that k is odd. In 1829 Bowditch, in his English translation, also noticed it, but he simply commented in a footnote, "Because the cosine of any uneven multiple of $1/2\,\pi$ is equal to nothing." It could be suggested that in his happiness to find a glitch in the work of such a famous mathematician, the still very young Hamilton wrote a bit boastful, calling it a "correction" instead of a "remark".

But turning the page in Graves' biography, the continuation of Hamilton's "correction" is a different proof, and that is a real surprise. In his proof Hamilton does not show what happens if $x = 0$, thus needing the comment that k is odd; he simply shows that if $k \neq 1$ there is a problem. If θ is not 0 or $1/2\,\pi$, the forces x and y are neither 0 nor infinite, but if $k \neq 1$ there is a θ for which $k\,\theta = 1/2\,\pi$. "Let θ be taken $= 1/2\,\pi\,/\,k$, and while x and y are real quantities, the formula $x = z\cos\,k\,\theta$ will give $x = 0$, which is absurd." He thus completely circumvents any need for extra remarks, and therefore can give his proof in a few sentences. And although calling Laplace's glitch an "error" still seems a bit overdone, this new proof is what he may have called his "correction".