Hamilton's quaternionic operator $\triangleleft$, now the vectorial nabla or del operator ∇,

which he developed in 1847.

The "well-known characteristic of operation," as Hamilton called it,

$$\left(\frac{\mathrm{d}}{\mathrm{d}x}\right)^2 + \left(\frac{\mathrm{d}}{\mathrm{d}y}\right)^2 + \left(\frac{\mathrm{d}}{\mathrm{d}z}\right)^2, \tag{1}$$

had been used by Laplace in 1799, and was defined in 1833 by Robert Murphy who gave it the symbol Δ. Hamilton introduced a "new characteristic of operation" $\triangleleft$, which he defined as

$$\triangleleft = \frac{i\,\mathrm{d}}{\mathrm{d}x} + \frac{j\,\mathrm{d}}{\mathrm{d}y} + \frac{k\,\mathrm{d}}{\mathrm{d}z}\,. \tag{2}$$

For its square, he found

$$-\triangleleft^2 = \left(\frac{\mathrm{d}}{\mathrm{d}x}\right)^2 + \left(\frac{\mathrm{d}}{\mathrm{d}y}\right)^2 + \left(\frac{\mathrm{d}}{\mathrm{d}z}\right)^2, \tag{3}$$

the minus sign indicating it was a quaternion operator, but Hamilton did not name it.

In 1870, Peter Guthrie Tait rotated the symbol $\triangleleft$ into ∇; in 1873 James Clerk Maxwell named ∇ "Hamilton's operator", and on the next page, he named (1) "Laplace's operator", giving it as

$$\nabla^2 = -\left(\frac{d^2}{dx^2} + \frac{d^2}{dy^2} + \frac{d^2}{dz^2}\right). \tag{4}$$

In 1890 Tait named the symbol ∇ "nabla," and in 1901, having been transformed into a vector operator in the modern sense, therefore the right-hand side of (4) had lost its minus sign while the right-hand side of (2) had remained the same, Edwin Bidwell Wilson named it "del".

As an "illustration" of (3), still in 1847 Hamilton gave the "known thermological equation", derived by Jean Baptiste Fourier in 1822, in which v is temperature,

$$\frac{\mathrm{d}^2v}{\mathrm{d}x^2} + \frac{\mathrm{d}^2v}{\mathrm{d}y^2} + \frac{\mathrm{d}^2v}{\mathrm{d}z^2} + a\frac{\mathrm{d}v}{\mathrm{d}t} = 0\,, \qquad \text{as} \qquad \left(\triangleleft^2 - \frac{a\,\mathrm{d}}{\mathrm{d}t}\right)v = 0, \tag{5}$$

"while $\triangleleft$ denotes, in quantity and in direction, the *flux* of heat, at the time t and at the point xyz." The heat equation is now usually written, see for instance pp. 188 and 582 of Vector Calculus, Marsden & Tromba 2003, as

$$k\left(\frac{\partial^2T}{\partial x^2} + \frac{\partial^2T}{\partial y^2} + \frac{\partial^2T}{\partial z^2}\right) = \frac{\partial T}{\partial t}, \qquad \text{and} \qquad \kappa\nabla^2T = \frac{\partial T}{\partial t}, \tag{6}$$

which, with T the temperature, is completely equivalent to (5).

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